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Unification of Type II Strings and T-duality

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We present a unified description of the low-energy limits of type II string theories. This is achieved by a formulation that doubles the space-time coordinates in order to realize the T-duality group $O(10,10)$ geometrically. The Ramond-Ramond fields are described by a spinor of $O(10,10)$, which couples to the gravitational fields via the Spin(10,10) representative of the so-called generalized metric. This theory, which is supplemented by a T-duality covariant self-duality constraint, unifies the type II theories in that each of them is obtained for a particular subspace of the doubled space.

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Superstring theory in ten dimensions is arguably the most promising candidate for a unified quantum mechanical description of gravity and other interactions. This theory, however, takes different guises. For instance, there are two different string theories with maximal supersymmetry, the type IIA and the type IIB theory. The ten-dimensional superstring theories, together with 11-dimensional supergravity, are different limits of a single underlying theory and are related through a web of dualities (see, e.g., [1]). The simplest of these dualities is T-duality that, for instance, relates type IIA string theory on the circular background $\mathbb{R}^{8,1}\times S^1$ of radius R to type IIB string theory on the same background, but with radius $1/R$.

In its low-energy limit string theory is described by Einstein's theory of general relativity, coupled to particular matter fields. In this description, T-duality results in the appearance of the hidden symmetry group $O(d,d)$ upon dimensional reduction on a torus T^d. Moreover, the low-energy limits of type IIA and type IIB give rise to the same theory, consistent with their equivalence under T-duality [2].

The general coordinate invariance of Einstein gravity naturally explains the presence of the $GL(d)$ subgroup, but the emergence of the full $O(d,d)$ upon dimensional reduction requires the precise matter couplings predicted by string theory, hinting at a novel geometrical structure. Recently, a 'double field theory' (DFT) has been found which realizes a T-duality group prior to dimensional reduction [3, 4] (see also [5, 6]). By doubling the space-time coordinates, the low-energy effective action of bosonic string theory or, equivalently, of the Neveu-Schwarz–Neveu-Schwarz (NS-NS) sector of superstring theory, can be extended to an action that has $O(D,D)$ as a global symmetry, where D is the space-time dimension.

In this Letter we introduce the extension to the Ramond-Ramond (RR) sector of type II strings, which will lead to a theory that contains all type II theories simultaneously in different T-duality 'frames'. Here we will not present explicit derivations, but a more detailed exposition will appear elsewhere [7]. Related work has appeared in [8, 9].

We start by reviewing the NS-NS subsector. It consists of the metric g_{ij}, the Kalb-Ramond 2-form b_{ij} and the scalar dilaton ϕ, where $i,j,\ldots=1,\ldots,D$ are space-time indices. The DFT is formulated in terms of a dilaton density d, which is related to ϕ via the field redefinition $e^{-2d}=\sqrt{g}e^{-2\phi}$, $g=|\det g|$, and the 'generalized metric'

$$\mathcal{H}_{MN}=\begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij}-b_{ik}g^{kl}b_{lj}\end{pmatrix}\,, \tag{1}$$

which combines g and b into an $O(D,D)$ covariant tensor with indices $M,N,\ldots=1,\ldots,2D$. All fields depend on the doubled coordinates $X^M=(\tilde{x}_i,x^i)$. We can regard $\mathcal{H}$ as the fundamental field, taking values in $SO(D,D)$ and satisfying $\mathcal{H}^T=\mathcal{H}$, and view (1) as just a particular parametrization. The action can be written as

$$S=\int dx\,d\tilde{x}\,e^{-2d}\,\mathcal{R}(\mathcal{H},d)\,, \tag{2}$$

where $\mathcal{R}(\mathcal{H},d)$ is an $O(D,D)$ invariant scalar, cf. (4.24) in the second reference of [4], and we use the short-hand notation $dx=d^Dx$. The action is invariant under the gauge transformations

$$\begin{aligned}\delta_\xi\mathcal{H}_{MN}&=\xi^P\partial_P\mathcal{H}_{MN}+2\big(\partial_{(M}\xi^P-\partial^P\xi_{(M}\big)\,\mathcal{H}_{N)P}\,,\\ \delta_\xi d&=\xi^M\partial_M d-\frac{1}{2}\partial_M\xi^M\,,\end{aligned} \tag{3}$$

with the derivatives $\partial_M=(\tilde{\partial}^i,\partial_i)$. Here, $O(D,D)$ indices M,N are raised and lowered with the invariant metric

$$\eta_{MN}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\,, \tag{4}$$

and (anti-)symmetrizations are accompanied by the combinatorial factor $\frac{1}{2}$. The consistency of the above theory requires the constraint

$$\partial^M\partial_M A=\eta^{MN}\partial_M\partial_N A=0\,,\quad \partial^M A\,\partial_M B=0\,, \tag{5}$$

for all fields and parameters A and B. This constraint implies that locally the fields depend only on half of the coordinates, and one can always find an $O(D,D)$ transformation into a frame in which the fields depend only

on the x^i. If one drops the dependence on the 'dual coordinates' $\tilde{x}_i$ in (2) or, equivalently, sets $\tilde{\partial}^i = 0$, the action reduces to the conventional low-energy effective action

$$S = \int d^D x \sqrt{g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} \right] , \quad (6)$$

where $H_{ijk} = 3\partial_{[i} b_{jk]}$ is the field strength of the 2-form. Moreover, for $\tilde{\partial}^i = 0$ the gauge transformations (3) with parameter $\xi^M = (\tilde{\xi}_i, \xi^i)$ reduce to the conventional general coordinate transformations $x^i \to x^i - \xi^i(x)$ and to the gauge transformations of the 2-form, $\delta b_{ij} = 2\partial_{[i}\tilde{\xi}_{j]}$.

Let us now turn to the extension by the RR sector. In this we make significant use of the work of Fukuma, Oota, and Tanaka [10]. (See also [11, 12].) The RR sector consists of forms of degrees 1 and 3 for type IIA and of degree 2 and 4 for type IIB, where the 5-form field strength of the 4-form is subject to a self-duality constraint. Here, we will use a democratic formulation that simultaneously uses dual forms, such that type IIA contains all odd forms, and type IIB contains all even forms, both being supplemented by duality relations [10]. The set of all forms naturally combines into a Majorana spinor of $O(10,10)$. Imposing an additional Weyl condition yields a spinor containing either all even or all odd forms, and we will show that the DFT extension of the RR sector can be formulated in terms of such a spinor.

We start by fixing our conventions for the spinor representation, setting $D = 10$ from now on. More precisely, these are representations of the double covering groups $\mathrm{Pin}(10,10)$ of $O(10,10)$, and $\mathrm{Spin}(10,10)$ of $SO(10,10)$. The gamma matrices satisfy the Clifford algebra

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN} \mathbf{1} . \quad (7)$$

A convenient representation can be constructed using fermionic oscillators ψ^i and ψ_i, satisfying

$$\{\psi_i, \psi^j\} = \delta_i{}^j , \quad \{\psi_i, \psi_j\} = 0 , \quad \{\psi^i, \psi^j\} = 0 , \quad (8)$$

where $(\psi_i)^\dagger = \psi^i$. With (4) we infer that they realize the algebra (7) via

$$\Gamma_i = \sqrt{2}\psi_i , \quad \Gamma^i = \sqrt{2}\psi^i . \quad (9)$$

Introducing a 'Clifford vacuum' $|0\rangle$ with $\psi_i|0\rangle = 0$ for all i, and the normalization $\langle 0|0\rangle = 1$, we can construct the representation by successive application of the raising operators ψ^i. A general spinor state then reads

$$\chi = \sum_{p=0}^{D} \frac{1}{p!} C_{i_1 \dots i_p} \psi^{i_1} \dots \psi^{i_p} |0\rangle , \quad (10)$$

whose coefficients $C_{i_1\dots i_p}$ can be identified with p-forms $C^{(p)}$. Any element S of the Pin group projects, via a group homomorphism $\rho : \mathrm{Pin}(10,10) \to O(10,10)$, to an element $h \in O(10,10)$,

$$S\,\Gamma_M\, S^{-1} = \Gamma_N\, h^N{}_M , \quad h = \rho(S) , \quad (11)$$

where $h\eta h^T = \eta$. Conversely, for any $h \in O(10,10)$, there is an $S \in \mathrm{Pin}(10,10)$ such that both $\pm S$ project to h. A spinor can be projected to a spinor of fixed chirality, i.e., to eigenstates $\chi_\pm$ of $(-1)^{N_F}$ with eigenvalues ± 1, where $N_F = \sum_k \psi^k \psi_k$ is the 'fermion number operator'. The spinor χ_+ of positive chirality then contains only even forms, and the spinor χ_- of negative chirality contains only odd forms. Imposing a chirality constraint reduces the symmetry from $\mathrm{Pin}(10,10)$ to $\mathrm{Spin}(10,10)$ since only the latter leaves this constraint invariant. Finally, we need the charge conjugation matrix satisfying

$$C\,\Gamma^M\, C^{-1} = (\Gamma^M)^\dagger . \quad (12)$$

A particular realization is given by

$$C = (\psi^1 - \psi_1)(\psi^2 - \psi_2)\cdots(\psi^{10} - \psi_{10}) , \quad (13)$$

which satisfies $C\psi_i C^{-1} = \psi^i$ and thereby (12).

Given a spinor (10) we can act with the Dirac operator

$$\partial\!\!\!/ \equiv \frac{1}{\sqrt{2}}\,\Gamma^M \partial_M = \psi^i \partial_i + \psi_i \tilde{\partial}^i , \quad (14)$$

which can be viewed as the $O(10,10)$ invariant extension of the exterior derivative d. In fact, for $\tilde{\partial} = 0$, it differentiates with respect to x^i and increases the form degree by one, thus acting like d. Moreover, it squares to zero,

$$\partial\!\!\!/^2 = \frac{1}{2}\Gamma^M \Gamma^N \partial_M \partial_N = \frac{1}{2}\eta^{MN}\partial_M \partial_N = 0 , \quad (15)$$

using (7) and the constraint (5).

In order to write an action that couples the NS-NS fields represented by the generalized metric $\mathcal{H}$ in (1) to the RR fields represented by a spinor χ, we note that the matrix $\mathcal{H}$ is an $SO(10,10)$ group element and thus has a representative in $\mathrm{Spin}(10,10)$, as has been used in dimensionally reduced theories [10]. In our case, however, a subtlety arises because (1) contains the full space-time metric, which we assume to be of Lorentzian signature. $SO(10,10)$ has two connected components, $SO^+(10,10)$, which contains the identity, and $SO^-(10,10)$. Due to the Lorentzian signature of g, $\mathcal{H}$ is actually an element of $SO^-(10,10)$. It turns out that a spin representative $S_{\mathcal{H}} \in \mathrm{Spin}(10,10)$ of $\mathcal{H}$ cannot be constructed consistently over the space of all $\mathcal{H}$. For instance, one may find a closed loop $\mathcal{H}(t)$, $t \in [0,1]$, $\mathcal{H}(0) = \mathcal{H}(1)$, in $SO^-(10,10)$, with the initial and final elements related by a *time-like* T-duality, for which a continuously defined spin representative yields $S_{\mathcal{H}(1)} = -S_{\mathcal{H}(0)}$. As a result, time-like T-dualities cannot be realized as transformations of the conventional fields g and b. Nevertheless, a fully T-duality invariant action can be written if we treat the spin representative itself as the dynamical field. We thus introduce a field $\mathbb{S}$, satisfying

$$\mathbb{S} = \mathbb{S}^\dagger , \qquad \mathbb{S} \in \mathrm{Spin}^-(10,10) . \quad (16)$$

The generalized metric is then *defined* by the group homomorphism, $\rho(\mathbb{S}) = \mathcal{H}$. By (16) and the general properties of the group homomorphism [7], $\mathcal{H}^T = \rho(\mathbb{S}^\dagger) = \mathcal{H}$ and so, as required, $\mathcal{H}$ is symmetric.

We are now ready to define the DFT formulation of type II theories, whose independent fields are $\mathbb{S}$, d and χ. The action reads

$$S = \int dx d\tilde{x} \left(e^{-2d}\,\mathcal{R}(\mathcal{H}, d) + \frac{1}{4}(\partial\!\!\!/\chi)^\dagger\,\mathbb{S}\,\partial\!\!\!/\chi \right) , \qquad (17)$$

and is supplemented by the self-duality constraint

$$\partial\!\!\!/\chi = -\mathcal{K}\,\partial\!\!\!/\chi\,, \quad \mathcal{K} \equiv C^{-1}\mathbb{S}\,. \qquad (18)$$

For the special case of type IIA, a similar duality relation has also been proposed in [8].

The field equation of χ reads

$$\partial\!\!\!/(\mathcal{K}\,\partial\!\!\!/\chi) = 0\,, \qquad (19)$$

which also follows as an integrability condition from the duality relation (18), upon acting with $\partial\!\!\!/$ and using (15). The field equation of $\mathbb{S}$ reads

$$\mathcal{R}_{MN} + \mathcal{E}_{MN} = 0\,, \qquad (20)$$

where $\mathcal{R}_{MN}$ is the DFT extension of the Ricci tensor [4], and the 'energy-momentum' tensor reads, using (18),

$$\mathcal{E}^{MN} = -\tfrac{1}{16}\mathcal{H}^{(M}{}_{P}\,\overline{\partial\!\!\!/\chi}\,\Gamma^{N)P}\,\partial\!\!\!/\chi\,. \qquad (21)$$

Let us now discuss the symmetries of this theory. First, it is invariant under a global action by $S \in \mathrm{Spin}^+(10,10)$,

$$\chi \to S\chi\,, \quad \mathbb{S} \to \mathbb{S}' = (S^{-1})^\dagger\,\mathbb{S}\,S^{-1}\,, \qquad (22)$$

implying $\partial\!\!\!/\chi \to S\partial\!\!\!/\chi$. Specifically, χ is assumed to have a fixed chirality, which breaks the invariance group of the action from $\mathrm{Pin}(10,10)$ to $\mathrm{Spin}(10,10)$, while the duality relations break the invariance group to $\mathrm{Spin}^+(10,10)$. The gauge symmetries of this theory are given by

$$\delta_\lambda\chi = \partial\!\!\!/\lambda\,, \qquad (23)$$

with spinorial parameter λ, leaving (17) and (18) manifestly invariant by (15), and the gauge symmetry (3) parametrized by ξ^M. On the new fields $\mathbb{S}$ and χ it reads

$$\begin{aligned} \delta_\xi\chi &= \xi^M\partial_M\chi + \frac{1}{2}\partial_M\xi_N\Gamma^M\Gamma^N\chi\,,\\ \delta_\xi\mathcal{K} &= \xi^M\partial_M\mathcal{K} + \frac{1}{2}\big[\Gamma^{PQ},\mathcal{K}\big]\partial_P\xi_Q\,, \end{aligned} \qquad (24)$$

where we have written the gauge variation of $\mathbb{S}$ in terms of $\mathcal{K}$ defined in (18). It can be checked that this gauge transformation gives rise to the required variation (3) of $\mathcal{H}$ upon application of ρ.

We will now evaluate the DFT defined by (17) and (18) in particular T-duality 'frames', starting with $\tilde{\partial}^i = 0$. To this end, we have to choose a particular parametrization of $\mathbb{S}$. Writing

$$\mathcal{H} = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}\begin{pmatrix} g^{-1} & 0 \\ 0 & g \end{pmatrix}\begin{pmatrix} 1 & -b \\ 0 & 1 \end{pmatrix} \equiv h_b^T\,h_g^{-1}\,h_b\,, \qquad (25)$$

we have to find spin representatives of the group elements h_b and h_g. The subtlety here is that, with g Lorentzian, h_g takes values in $SO^-(10,10)$ and thus is not in the component connected to the identity. It is then convenient to write g in terms of vielbeins,

$$g = e\,k\,e^T\,, \quad h_g = h_e h_k h_e^T\,, \qquad (26)$$

where e has positive determinant, i.e., $e \in GL^+(10)$, and k is the flat Minkowski metric $\mathrm{diag}(-1,1,\ldots,1)$. The group elements h_e and h_b are in the component connected to the identity and so their spin representatives can be written as simple exponentials,

$$S_b = e^{-\frac{1}{2}b_{ij}\psi^i\psi^j}\,, \quad S_e = \frac{1}{\sqrt{\det e}}\,e^{\psi^i E_i{}^j\psi_j}\,, \qquad (27)$$

with $e = \exp(E)$, as can be verified with (11). A spin representative for the matrix k can be chosen to be [13]

$$S_k = \psi^1\psi_1 - \psi_1\psi^1\,, \qquad (28)$$

where 1 labels the time-like coordinate. This can also be verified with (11). A spin representative $S_{\mathcal{H}}$ of $\mathcal{H}$ can then locally be defined as

$$S_{\mathcal{H}} \equiv S_b^\dagger\,S_g^{-1}\,S_b\,, \quad S_g = S_e S_k S_e^\dagger\,. \qquad (29)$$

We now set $\mathbb{S} = S_{\mathcal{H}}$, but we stress that this is just a particular parameterization in much the same way that (1) is just a particular parametrization of $\mathcal{H}$.

It is now straightforward to evaluate the action (17) for $\tilde{\partial} = 0$. First, as noted above, $\partial\!\!\!/\chi$ reduces to the exterior derivatives of the $C^{(p)}$, $F^{(p+1)} \equiv dC^{(p)}$. The action of S_b in $S_{\mathcal{H}}$ then modifies this, using (27), to

$$\widehat{F} = e^{-b^{(2)}} \wedge F = e^{-b^{(2)}} \wedge dC\,. \qquad (30)$$

Second, (29) implies for the action of S_g^{-1}

$$S_g^{-1}\psi^{i_1}\ldots\psi^{i_p}|0\rangle = -\sqrt{g}g^{i_1j_1}\ldots g^{i_pj_p}\psi^{j_1}\ldots\psi^{j_p}|0\rangle\,. \qquad (31)$$

The Lagrangian corresponding to the RR part of (17) then reduces to kinetic terms for all forms,

$$\mathcal{L}_{\mathrm{RR}} = -\frac{1}{4}\sqrt{g}\sum_{p=1}^{D}\frac{1}{p!}\,g^{i_1j_1}\cdots g^{i_pj_p}\,\widehat{F}_{i_1\ldots i_p}\widehat{F}_{j_1\ldots j_p}\,, \qquad (32)$$

where we recall that the sum extends over all even or all odd forms, depending on the chirality of χ. Similarly, using (13), the self-duality constraint (18) reduces to the conventional duality relations (with the Hodge star $*$),

$$\widehat{F}^{(p)} = (-1)^{\frac{(D-p)(D-p-1)}{2}} * \widehat{F}^{(D-p)}\,. \qquad (33)$$

We have thus obtained the democratic formulation of type II theories, whose field equations are equivalent to the conventional field equations of type IIA for odd forms and of type IIB for even forms [10].

Let us briefly comment on the gauge symmetries for $\tilde{\partial} = 0$. The transformations (24) for χ, parameterized by $\xi^M = (\tilde{\xi}_i, \xi^i)$, reduce to the conventional general coordinate transformations $x^i \to x^i - \xi^i(x)$ of the p-forms $C^{(p)}$, but also to non-trivial transformations under the b-field gauge parameter $\tilde{\xi}_i$, $\delta_{\tilde{\xi}} C = d\tilde{\xi} \wedge C$.

We turn now to the discussion of other T-duality frames, starting with $\partial_i = 0$, $\tilde{\partial}^i \neq 0$. For the analysis of this case it is convenient to perform a field redefinition according to the T-duality transformation J that exchanges x^i and $\tilde{x}_i$ and which, as a matrix, coincides with η defined in (4),

$$\mathcal{H}' \equiv J\,\mathcal{H}\,J = \mathcal{H}^{-1}\,. \tag{34}$$

It has been shown in [4] that the NS-NS part of the DFT reduces for $\partial_i = 0$ to the same action (6), but written in terms of the primed (T-dual) variables. Next, we define a corresponding field redefinition for the RR fields, using a spin representative S_J of J,

$$\chi' = S_J \chi\,, \quad \not{\partial}' = \psi^i \tilde{\partial}^i + \psi_i \partial_i\,, \quad \not{\partial}' \chi' = S_J \not{\partial} \chi\,. \tag{35}$$

For the RR action we then find

$$\begin{aligned}\mathcal{L}_{\rm RR} &= \tfrac{1}{4}(\not{\partial}\chi)^\dagger S_{\mathcal{H}} \not{\partial}\chi = \tfrac{1}{4}(\not{\partial}'\chi')^\dagger (S_J^{-1})^\dagger S_{\mathcal{H}} S_J^{-1} \not{\partial}'\chi' \\ &= -\tfrac{1}{4}(\not{\partial}'\chi')^\dagger S_{\mathcal{H}'} \not{\partial}'\chi'\,,\end{aligned} \tag{36}$$

where we used that J contains a time-like T-duality such that, as mentioned above, this leads to a sign factor in the transformation of $S_{\mathcal{H}}$. Thus, in the new variables the action takes the same form as in the original variables, up to a sign. The transformed Dirac operator in (35) implies that setting $\partial_i = 0$ in the first form in (36) is equivalent to setting $\not{\partial}' = \psi^i \tilde{\partial}^i$ in the final form in (36). This way to evaluate the action is, however, equivalent to our computation above of setting $\tilde{\partial} = 0$ in the original action, just with fields and derivatives replaced by primed fields and derivatives. Thus, we conclude that the DFT action reduces for $\partial_i = 0$ to a type II theory with the overall sign of the RR action reversed. These are known as type II* theories and have been introduced by Hull in the context of *time-like* T-duality [14]. They are defined such that the time-like circle reductions of type IIA (IIB) and type IIB* (IIA*) are equivalent. This result also implies that the overall sign of $\mathbb{S}$ has no physical significance in that it merely determines for which coordinates (x or $\tilde{x}$) we obtain the type II or type II* theory.

More generally, one finds that evaluating the DFT in a T-duality frame that is obtained by an odd (even) number of T-duality inversions from a frame in which the theory reduces, say, to type IIA, it reduces to the T-dual theory, i.e., type IIB (IIA) for space-like transformations and IIB* (IIA*) for time-like transformations. Summarizing, the DFT defined by (17) and (18) combines all type II theories in a single universal formulation. We hope that this theory may provide insights into the still elusive formulation of string theory as, e.g., for a yet to be constructed type II string field theory.

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